NTT Software Innovation Center

Fast Algorithm for Modularity-based Graph Clustering

Hiroaki Shiokawa
NTT Software Innovation Center, NTT Corporation,
July 23rd , 2013

BACKGROUND & MOTIVATION

Large Graphs

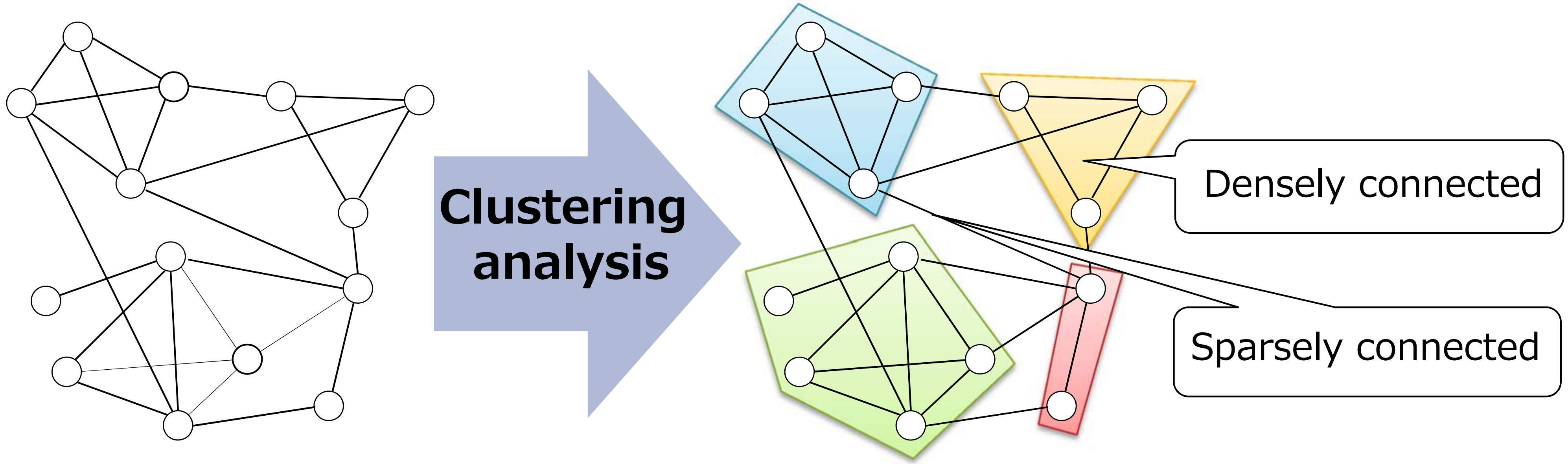
- **Large-scale graphs become available**
  - **Facebook**: **1.11 billion** active users / month(*1)
  - **Twitter**: **140 million active users** / day
    340 million new posts / day (*2)
  - And more …

- **A lot of techniques for analyzing massive-scale graph**
  - Massive data require so much time for analysis ☹
  - It is important to analyze large scale data quickly ☺

(*1) "Key Facts", http://newsroom.fb.com/Key-Facts
(*2) http://dev.twitter.com/media/authors

Graph Clustering

- **Graph clustering is one of the most important methods**
  - Community detection over social networks
  - Event detection from microblogging services
  - Brain Analysis, Image segmentation, …



Modularity-based Graph Clustering

- Clustering methods which find the division of graph to maximize the modularity measure
- Improvement of clustering speed

Our research target

1B ～ 100M nodes/hour	There are no algorithms ☹
10M nodes/hour	BGLL[*Blondel et al., 2008*]
1M nodes/hour	CNM[*Clauset et al., 2004*], WT[*Wakita et al., 2008*]
100k nodes/hour	Newman method [*Newman et al., 2004*]
10k nodes/hour	Girvan-Newman method [*Girvan et al., 2004*]

Objective and Contributions

- **Objective**

Fast graph clustering method with high modularity

- **3 key techniques**
  1. Incremental nodes aggregation
  2. Incremental nodes pruning
  3. Efficient ordering of nodes selections
- **Contributions of our algorithm**
  - **Efficiency**
    - Considerably faster than BGLL
    - Clusters 100M nodes within 3 minutes
  - **High Modularity**
    - Scores high modularity as same as BGLL
  - **Effectiveness**
    - Improves performances for complex networks

PRELIMINARIES

Modularity

- **Modularity evaluates the strength of division of a graph into clusters** [*Newman and Girvan 2004*]
  - Finding the division which maximizes modularity is NP-complete

    ⇒ A lot of greedy approaches were proposed

$$Q = \sum_{i \in C} \left\{ \underbrace{\frac{e_{ii}}{2m}}_{\text{The fraction of the edges within cluster } i} - \underbrace{\left(\frac{\sum_{j \in C} e_{ij}}{2m} \right)^2}_{\text{The fraction of outgoing edges from a cluster } i} \right\}$$

C : Set of cluster
e_{ij} : Number of edges between cluster i, j
m : Total number of edges in a graph

State of the art algorithm: BGLL

- **Continuing following passes until the modularity score is maximized**

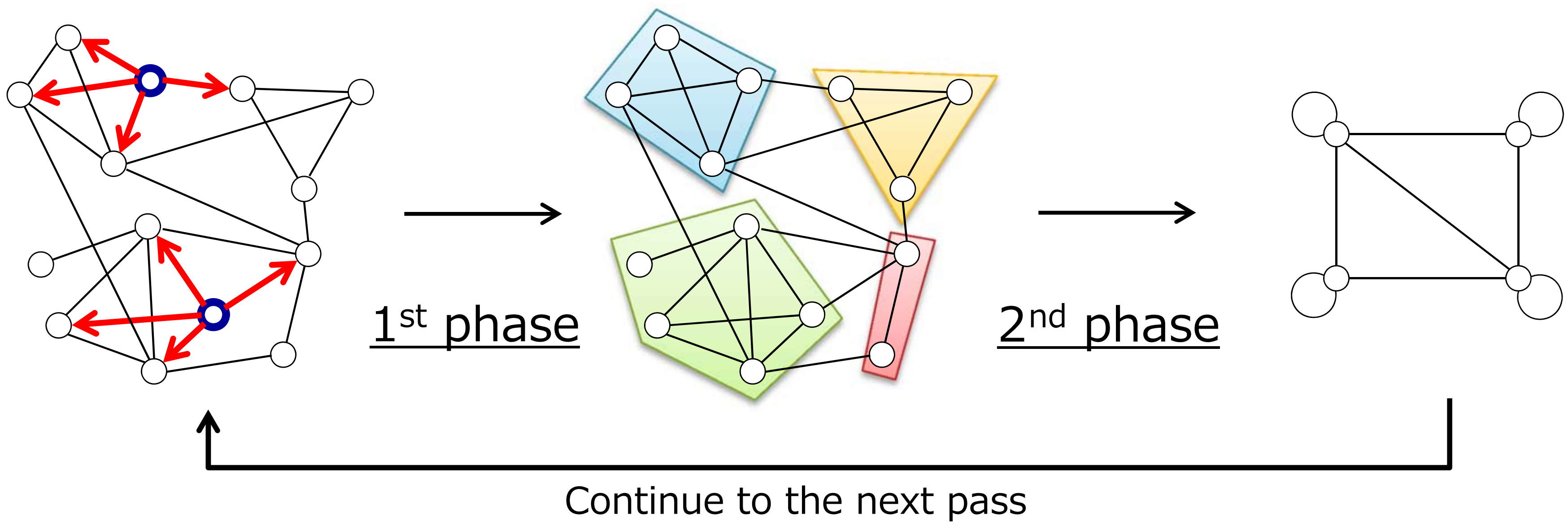
Pass =

1st phase: Local clustering

1) **Selects a node**
2) **Computes the modularity gain**
3) **Places the neighbor node** in the same cluster

2nd phase: Data reduction

- **Aggregates all nodes in the same cluster as a single node**



PROPOSED ALGORITHM

Overview of proposed algorithm

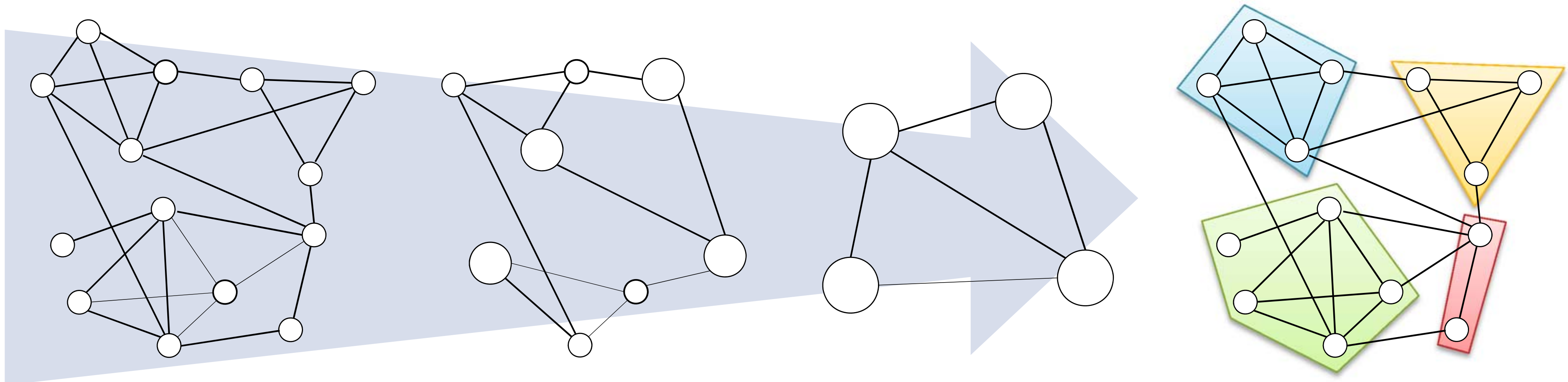
Our method

- 3 key techniques
  - Incremental aggregation
  - Incremental pruning
  - Efficient ordering

- Clustering coefficient
- Power-law degree distribution

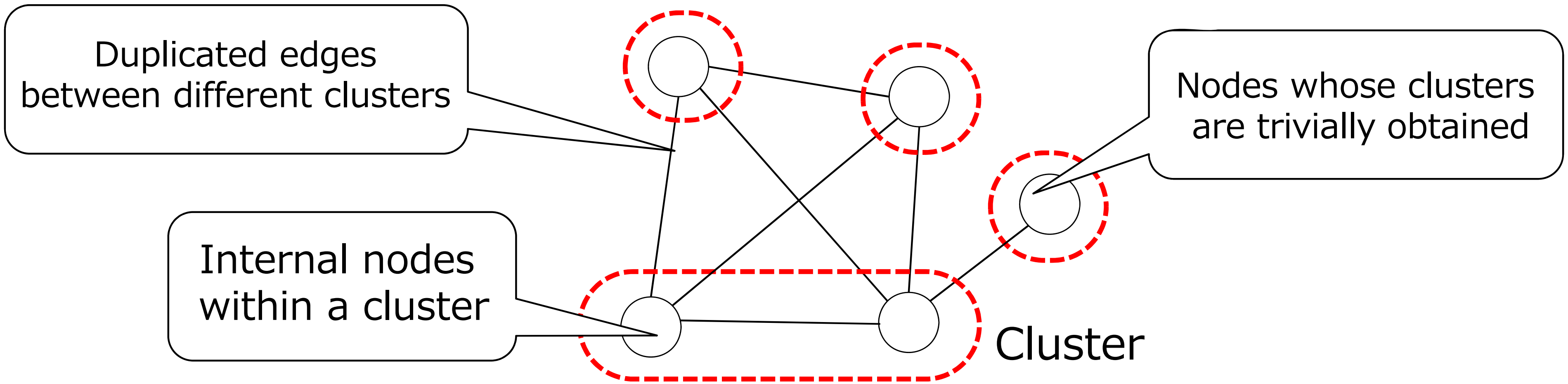
BGLL

- Batch based aggregation
- Random ordering



Idea 1 : Clustering coefficient

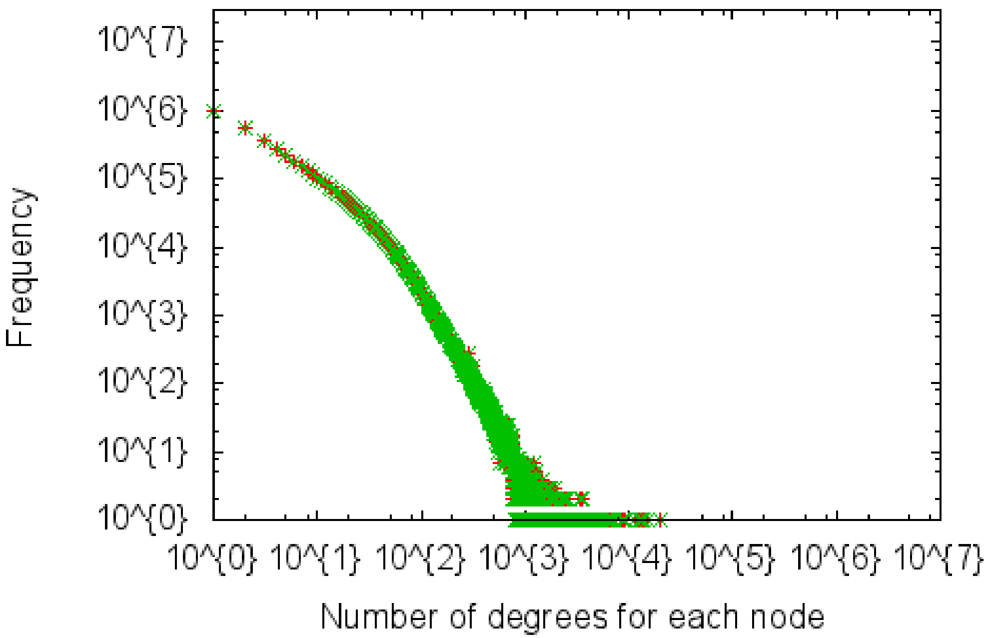
- **Complex networks have large clustering coefficient**
  - Clustering coefficient is a measure of degree to which nodes in a graph tend to cluster together
  - **There are many duplicated nodes/edges** in a graph which has large clustering coefficient



Idea 2 : Power-law degree distribution

- **Complex networks have highly skewed degree distribution following the power-law distribution**
  - Most of nodes only have a few neighbor nodes, and only few nodes have large number of neighbor nodes
  - The frequency *F* of nodes with degree *d* is $\boldsymbol{F \propto d^{-\alpha}}$

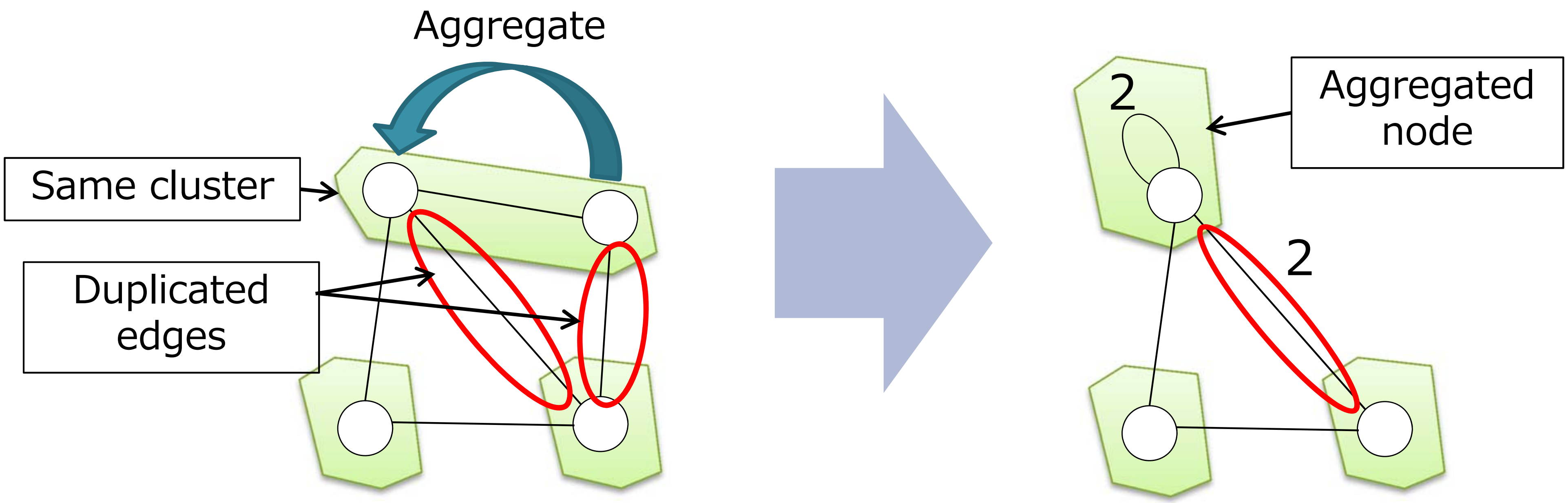
Example of degree distribution of complex network



- Random ordering of node selection leads redundant computation

Incremental Aggregation

- **Incrementally aggregate nodes which belong to the same cluster**
  - It aggregates duplicated edges between clusters while keeping the graph semantics
  - *Example*)



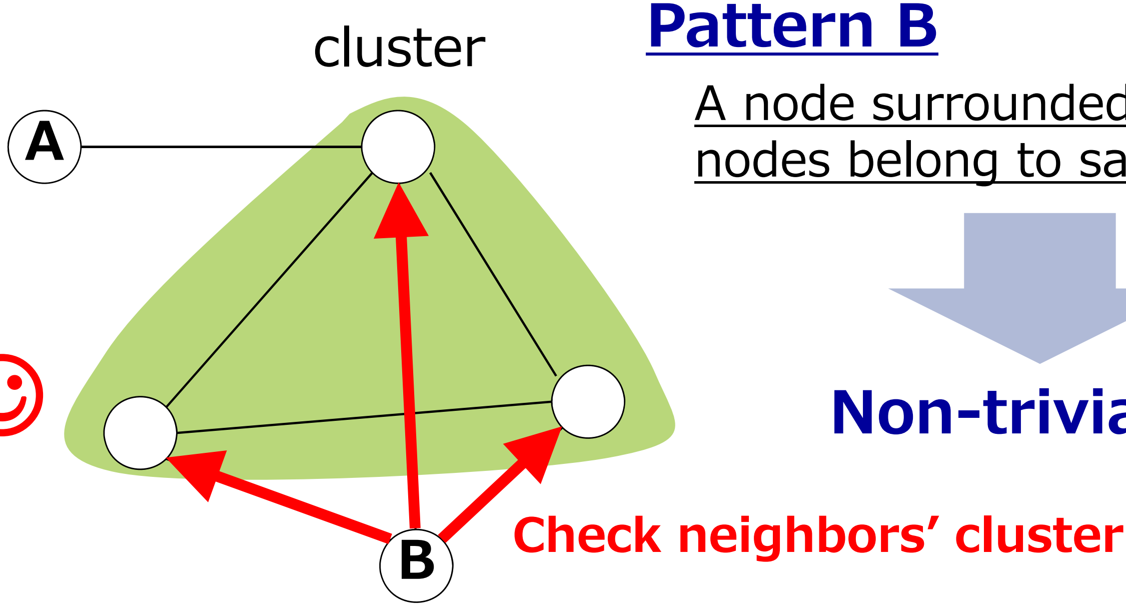
Incremental pruning

- **Incrementally prune nodes whose cluster is trivially obtained**
  - We can easily assign nodes to clusters without computing modularity gains
  - From the graph structure, there are **2 patterns of pruning**

Pattern A

A node only has a single neighbor node

Easy to prune nodes ☺



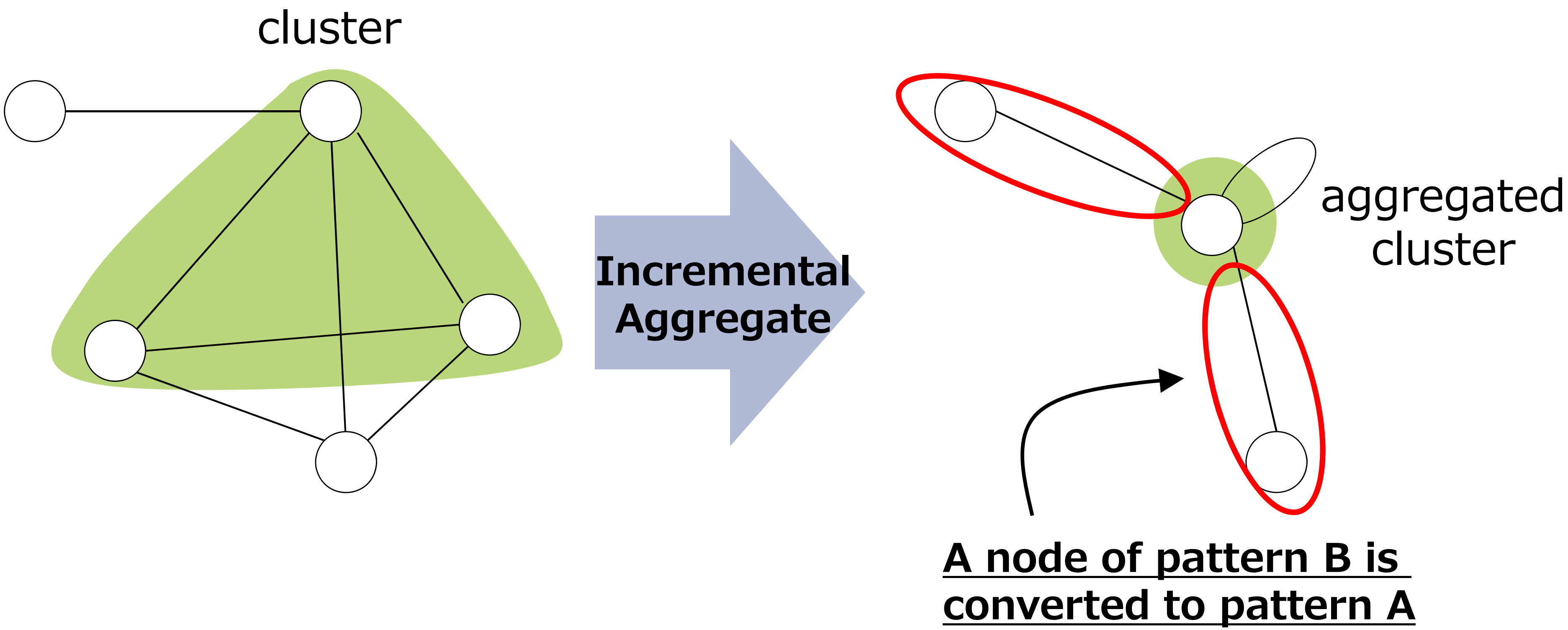
Pattern B

A node surrounded by nodes belong to same cluster

Non-trivial ☹

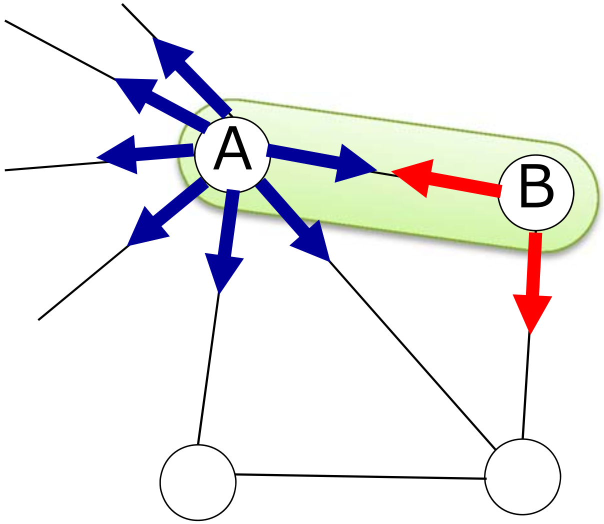
Incremental pruning (Cont.)

- **Efficient pruning approach for pattern 2**
  - All nodes within the same cluster have been aggregated to a node by incremental aggregation
  - We can find all prunable nodes by obtaining nodes such that they have only a single adjacent node



Efficient ordering of node selection

- **Dynamically selects a node with the smallest degree**
  - *Example*) Node A and B being assigned to the same cluster



Select from A
many computations
Select from B
2 computations

It's more efficient to compute from node B than node A

- **By selecting node with the smallest degree, we can avoid producing super-cluster structures**

EVALUATION

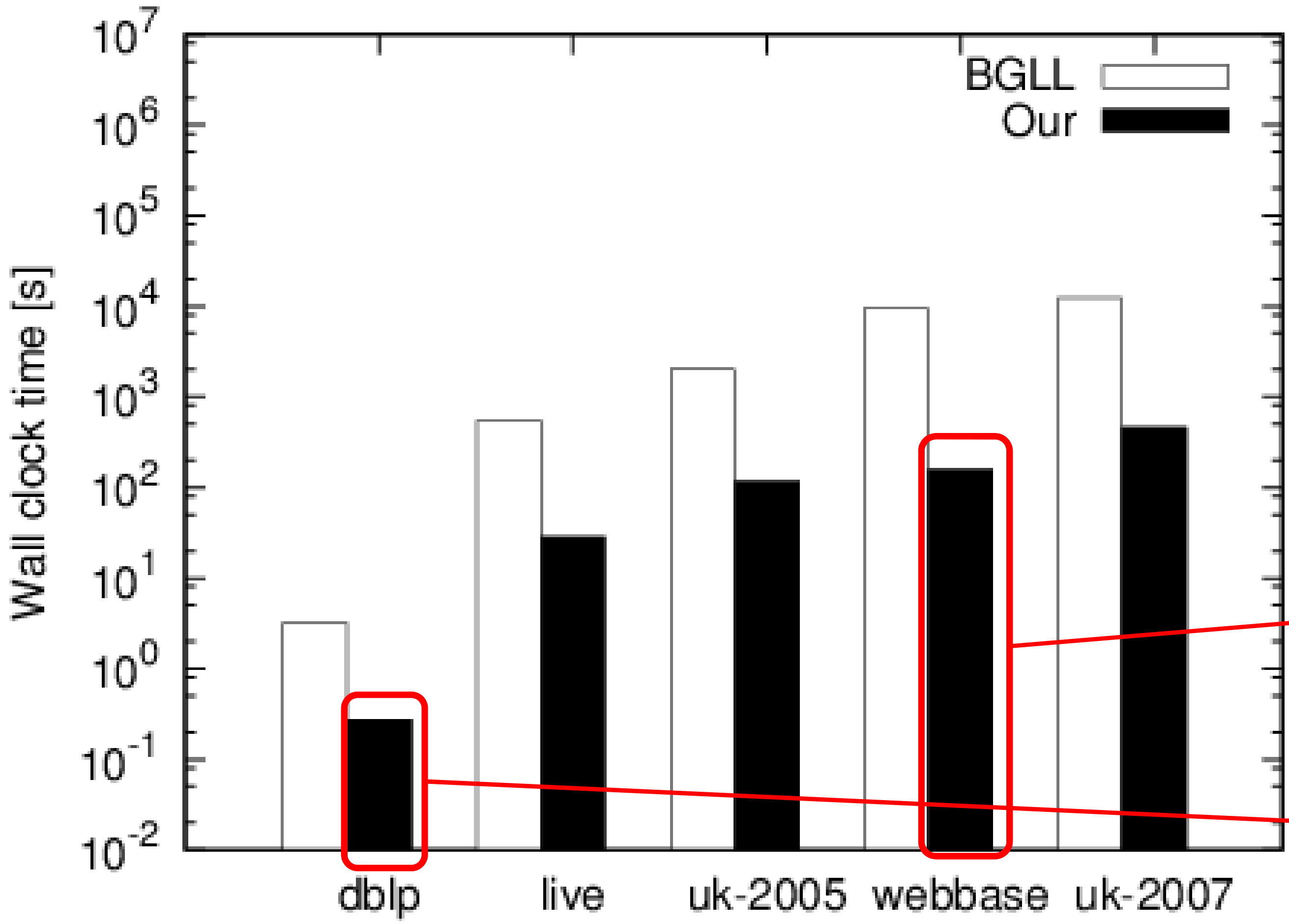
Datasets & Experimental Environment

- **Real world datasets**
  - 2 Social networks and 3 Web graphs of IP domains

Dataset	$\|V\|$	$\|E\|$	Skewness of degree distribution α
dblp-2010	326,186	1,615,400	2.82
ljournal-2008	5,363,260	79,023,142	2.29
uk-2005	39,459,925	936,364,282	1.71
webbase	118,142,155	1,019,903,190	2.14
uk2007-05	105,896,555	3,738,733,648	1.51

- **Experimental Environment**
  - All experiments were conducted on a Linux 2.6.18 server with Intel Xeon CPU L5640 2.27GHz and 144GB RAM
  - Run all methods on 1 core, 1CPU

Computational time



- Proposed is up to **60 times faster** than the state of the art algorithm BGLL
- Graphs with highly skewed degree distribution run faster than the other datasets

100 million nodes and 1 billion edges in 156 seconds!

320k nodes within 1 seconds!

Computational time – power-law differences

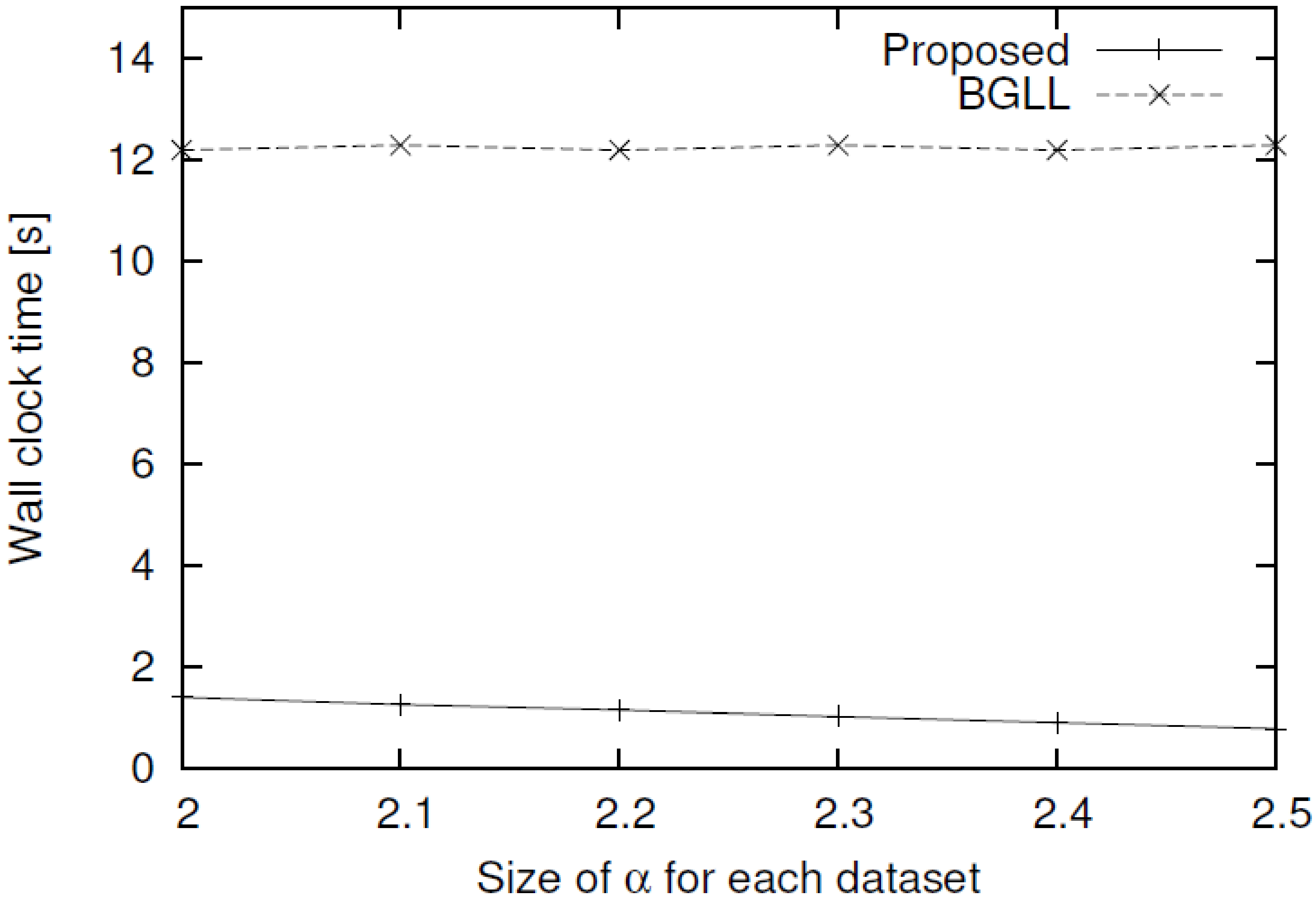


Figure 2: Power-law difference

Computational time – size differences

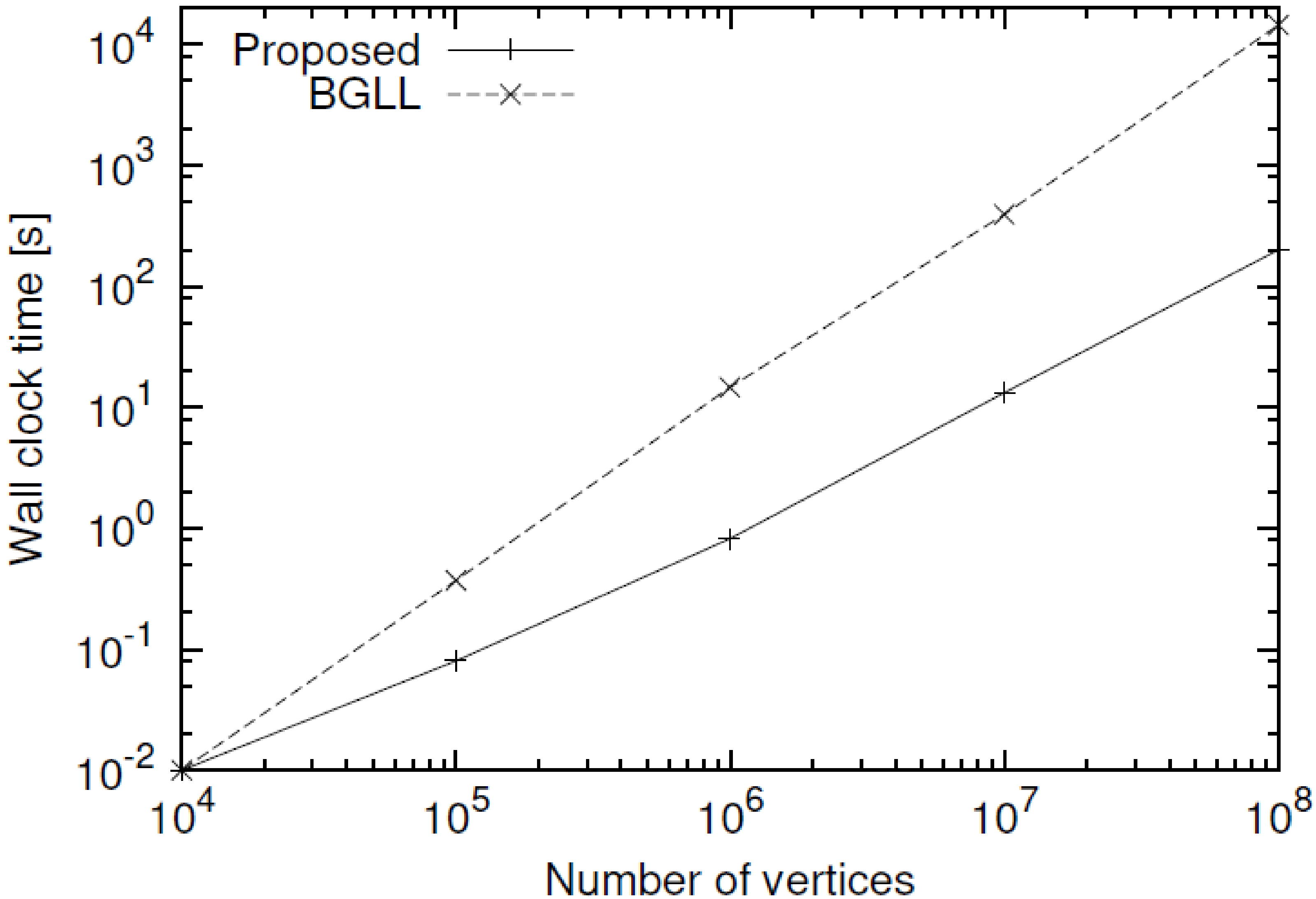


Figure 3: Scalability

Modularity score

- **Modularity score for datasets**
  - Large modularity score means the output of algorithms is well clustered
  - Proposed method achieves **almost same modularity scores as/slightly higher than BGLL**

Table 2: Modularity Q

	dblp-2010	ljournal-2008	uk-2005	webbase-2001	uk-2007-05
Proposed	0.90	0.74	0.98	0.98	0.97
BGLL	0.88	0.74	0.97	0.96	0.97

CONCLUSION

Conclusion

- **Fast clustering algorithm for large graphs**
  - **Our solution**
    - Incremental aggregation
    - Incremental pruning
    - Efficient ordering of nodes selections

- **Contribution of our algorithm**
  - **Efficiency**
    - Considerably faster than BGLL
    - Clusters 100M nodes within 3 minutes
  - **High Modularity**
    - Scores high modularity as same as BGLL
  - **Effectiveness**
    - Improves performance for complex networks