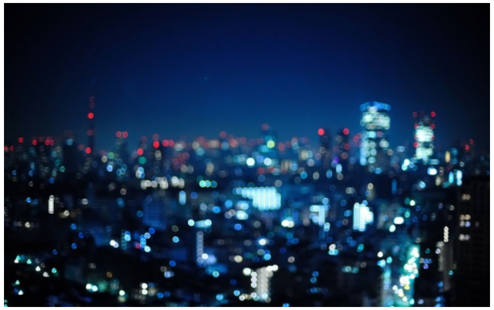
Distributed Private Data Collection at Scale

Graham Cormode

g.cormode@warwick.ac.uk

Tejas Kulkarni (Warwick)

Divesh Srivastava (AT&T)



THE UNIVERSITY OF WARWICK

Big data, big problem?

- The big data meme has taken root
  - Organizations jumped on the bandwagon
  - Entered the public vocabulary
- But this data is mostly about individuals
  - Individuals want privacy for their data
  - How can researchers work on sensitive data?
- The easy answer: anonymize it and share
- The problem: we don’t know how to do this

Data Release Horror Stories

Commonwealth of Massachusetts Group Insurance Commission

Your Benefits Connection

NETFLIX

AOL

NYC Taxi & Limousine Commission

We need to solve this data release problem...

THE UNIVERSITY OF WARWICK

Differential Privacy (Dwork et al 06)

A randomized algorithm K satisfies ε-differential privacy (DP) if:

Given two data sets that differ by one individual, D and D', and any property S:

$$\Pr[\,K(D) \in S] \;\leq\; e^{\varepsilon} \Pr[\,K(D') \in S]$$

- Can achieve DP for counts by adding a random noise value
- Uncertainty "hides" whether someone is present in the data
- Slowly being adopted in practice (e.g. US Census 2020)

THE UNIVERSITY OF
WARWICK

Privacy with a coin toss

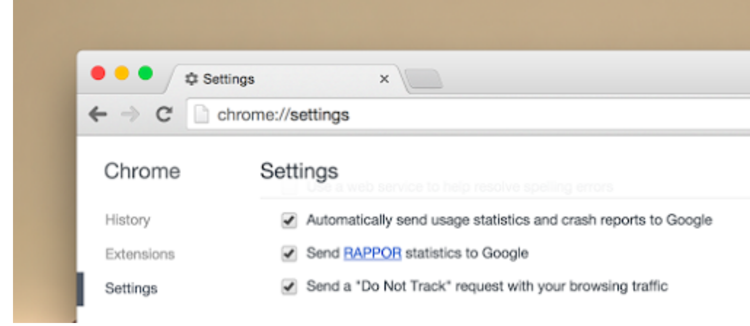
Perhaps the simplest possible DP algorithm

- Each user has a single private bit of information
  - Encoding e.g. political/sexual/religious preference, illness, etc.
- Toss a (biased) coin
  - With probability $p > ½$, report the true answer
  - With probability $1-p$, lie
- Collect the responses from a large number N of users
  - Can 'unbias' the estimate (if we know p) of the population fraction
  - The error in the estimate is proportional to $1/\sqrt{N}$
- Gives differential privacy with parameter $\varepsilon = \ln (p/(1-p))$
  - Works well in theory, but would anyone ever use this?

Privacy in practice

- Differential privacy based on coin tossing is widely deployed
  - In Google Chrome browser, to collect browsing statistics
  - In Apple iOS and MacOS, to collect typing statistics
  - By Snap(chat) to instantiate machine learning models
  - This yields deployments of over 100 million users
- The model where users apply differential privately and then aggregated is known as "Local Differential Privacy"
  - The alternative is to give data to a third party to aggregate
  - The coin tossing method is known as 'randomized response'
- Local Differential privacy is state of the art in 2019: Randomized response invented in 1965: five decade lead time!

RAPPOR: Bits with a twist

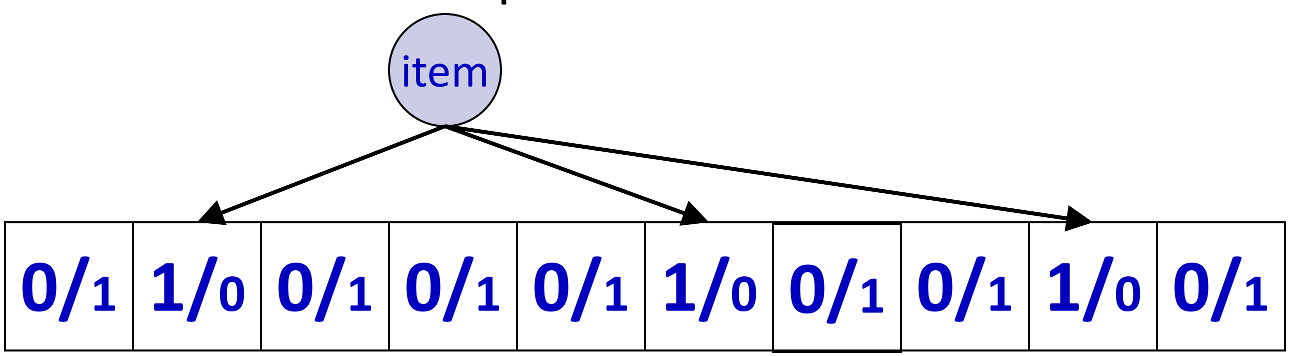


- Each user has one value out of a very large set of possibilities
  - E.g. their favourite URL, www.bbc.co.uk
- First attempt: run randomized response for all possible values
  - Do you have google.com? Nytimes.com? Bing.com? Bbc.co.uk?...
- Meets required privacy guarantees with parameter $2 \ln(p/(1-p))$
  - If we change a user's choice, then at most two bits change: a 1 goes to 0 and a 0 goes to 1
- Slow: sends 1 bit for every possible choice
  - And limited: can't easily handle new options being added
- Try to do better by reducing domain size through hashing

THE UNIVERSITY OF WARWICK

Bloom Filters + Randomized Response

- Idea: apply Randomized response to the bits in a Bloom filter
  - Not too many bits in the filter compared to all possibilities
- Each user maps their input to at most k bits in the filter
  - New choices can be counted (by hashing their identities)
- Privacy guarantee with parameter k ln (p/(1-p))
  - Combine all user reports and observe how often each bit is set



THE UNIVERSITY OF WARWICK

Decoding noisy Bloom filters

- We obtain a Bloom filter, where each bit is now a probability
- To estimate the frequency of a particular value:
  - Look up its bit locations in the Bloom filter
  - Compute the unbiased estimate of the probability each is 1
  - Take the minimum of these estimates as the frequency
- More advanced decoding heuristics to decode all at once
- How to find frequent strings without knowing them in advance?
  - Subsequent work: build up frequent strings character by character (using statistics on character co-occurrences)

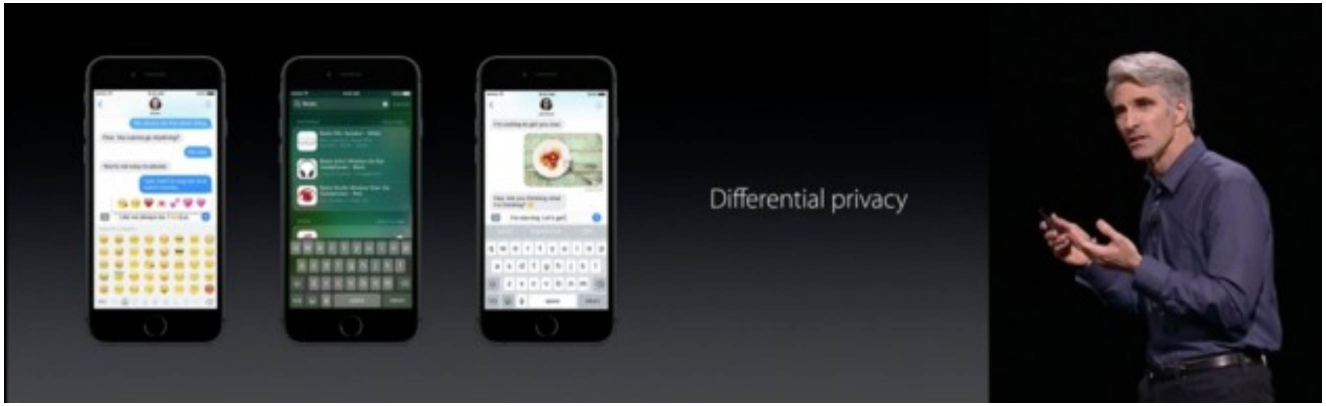
THE UNIVERSITY OF WARWICK

Rappor in practice

- The Rappor approach was implemented in the Chrome browser
  - Collects data from opt-in users, tens of millions per day
  - Open source implementation available
- Tracks settings in the browser (e.g. home page, search engine)
  - Identify if many users unexpectedly change their home page (indicative of malware)
- Typical configuration:
  - 128 bit Bloom filter, 2 hash functions, privacy parameter ~0.5
  - Needs about 10K reports to identify a value with confidence

Apple: sketches and transforms

- Similar problem to Rappor:
  want to count frequencies of many possible items
  - For simplicity, assume each user holds a single item
  - Want to reduce the burden of collection:
    can we further reduce the size of the summary?
- Instead of Bloom Filter, make use of sketches [C, Muthukrishnan 04]
  - Similar idea, but better suited to capturing frequencies



Count-Min Sketch + Randomized Response

- Each user encodes their (unit) input with a Count-Min sketch
  - Then applies randomized response to each entry
- Aggregator adds up all received sketches, unbiases the entries
- Take an unbiased estimate from the sketch based on mean
  - More robust than taking min when there is random noise
- Can bound the accuracy in the estimate via variance computation
  - Error is a random variable with variance proportional to $\|x\|_2^2/(sn)$
  - I.e. (absolute) error decreases proportional to $1/\sqrt{n}$, $1/\sqrt{\text{sketch size}}$
- Bigger sketch size s → more accuracy
  - But we want smaller communication?

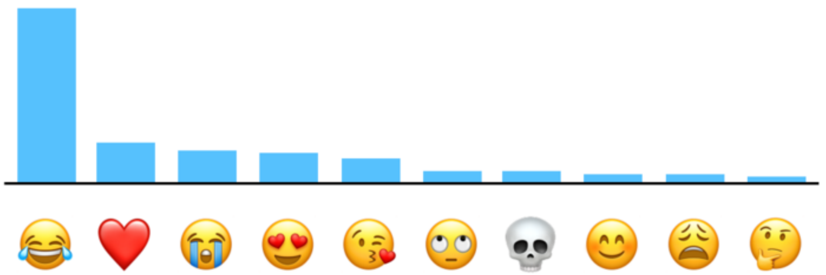
One weird trick: Hadamard transform

- The distribution of interest could be sparse and spiky
  - This is preserved under sketching
  - If we don’t report the whole sketch, we might lose information
- Idea: transform the data to ‘spread out’ the signal
  - Hadmard transform is a discrete Fourier transform
  - We will transform the sketched data
- Aggregator reconstructs the transformed sketch
  - Can invert the transform to get the sketch back
- Now the user just samples one entry in the transformed sketch
  - No danger of missing the important information – it’s everywhere
  - Variance is essentially unchanged from previous case
- User only has to send one bit of information

$$\begin{bmatrix} H^* & H^* \\ H^* & -H^* \end{bmatrix} = \begin{bmatrix} -1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 & 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 & 1 & 1 & 1 & -1 \\ -1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 \\ 1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 \\ 1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 \\ 1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 \end{bmatrix}$$

Apple’s Differential Privacy in Practice

- Apple use their system to collect data from iOS and OSX users
  - Popular emjois: (heart) (laugh) (smile) (crying) (sadface)
  - “New” words: bruh, hun, bae, tryna, despacito, mayweather
  - Which websites to mute, which to autoplay audio on!
  - Which websites use the most energy to render
- Deployment settings:
  - Sketch size w=1000, d=1000
  - Number of users not stated
  - Privacy parameter 2-8 (some criticism of this)



The Count Mean Sketch technique allows Apple to determine the most popular emoji to help design better ways to find and use our favorite emoji. The top emoji for US English speakers contained some surprising favorites.

THE UNIVERSITY OF WARWICK

Going beyond counts of data

- Simple frequencies can tell you a lot, but can we do more?
- Our work [SIGMOD18]: materializing marginal distributions
  - Each user has d bits of data (encoding sensitive data)
  - We are interested in the distribution of combinations of attributes

	Gender	Obese	High BP	Smoke	Disease
Alice	1	0	0	1	0
Bob	0	1	0	1	1
…					
Zayn	0	0	1	0	0

Gender/Obese	0	1
0	0.28	0.22
1	0.29	0.21

Disease/Smoke	0	1
0	0.55	0.15
1	0.10	0.20

Nail, meet hammer

- Could apply Randomized Reponse to each entry of each marginal
  - To give an overall guarantee of privacy, need to change p
  - The more bits released by a user, the closer p gets to ½ (noise)
- Need to design algorithms that minimize information per user
- First observation: the sampling trick
  - If we release n bits of information per user, the error is $n/\sqrt{N}$
  - If we sample 1 out of n bits, the error is $\sqrt{(n/N)}$
  - Quadratically better to sample than to share!

What to materialize?

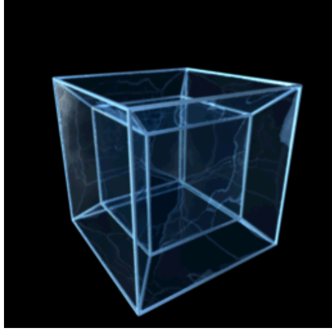
Different approaches based on how information is revealed

1. We could reveal information about all marginals of size k
   - There are $(d \text{ choose } k)$ such marginals, of size 2^k each
2. Or we could reveal information about the full distribution
   - There are 2^d entries in the d-dimensional distribution
   - Then aggregate results here (obtaining additional error)

- Still using randomized response on each entry
  - Approach 1 (marginals): cost proportional to $2^{3k/2} d^{k/2}/\sqrt{N}$
  - Approach 2 (full): cost proportional to $2^{(d+k)/2}/\sqrt{N}$
- If k is small (say, 2), and d is large (say 10s), Approach 1 is better
  - But there's another approach to try…

THE UNIVERSITY OF WARWICK

Hadamard transform (again)



Instead of materializing the data, we can transform it

- The Hadamard transform is the discrete Fourier transform for the binary hypercube
  - Very simple in practice

$$\begin{bmatrix} H^* & H^* \\ H^* & -H^* \end{bmatrix} = \begin{bmatrix} -1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 & 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 & 1 & 1 & 1 & -1 \\ -1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 \\ 1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 \\ 1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 \\ 1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 \end{bmatrix}.$$

- Property 1: only (d choose k) coefficients are needed to build any k-way marginal
  - Reduces the amount of information to release
- Property 2: Hadamard transform is a linear transform
  - Can estimate global coefficients by sampling and averaging
- Yields error proportional to $2^{k/2}d^{k/2}/\sqrt{N}$
  - Better than both previous methods (in theory)

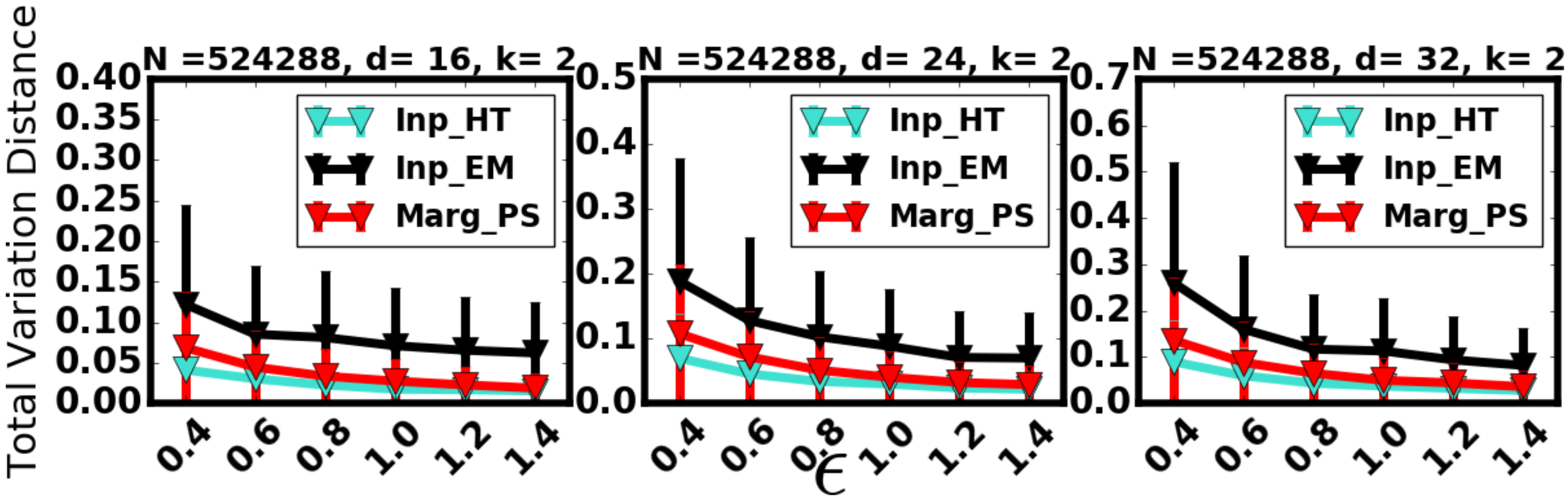
THE UNIVERSITY OF WARWICK

Outline of error bounds

How to prove these error bounds?

- Create a random variable X_i encoding the error from each user
  - Show that it is unbiased: $E[X_i]=0$, error is zero in expectation
- Compute a bound for its variance, $E[X_i^2]$ (including sampling)
- Use appropriate inequality to bound error of sum, $|\sum_{i=1}^N X_i|$
  - Bernstein or Hoeffding in equalities: error like $\sqrt{(N/E[X_i^2])}$
  - Typically, error in average of N goes as $1/\sqrt{N}$
- Possibly, second round of bounding error for further aggregation
  - E.g. first bound error to reconstruct full distribution, then error when aggregating to get a target marginal distribution

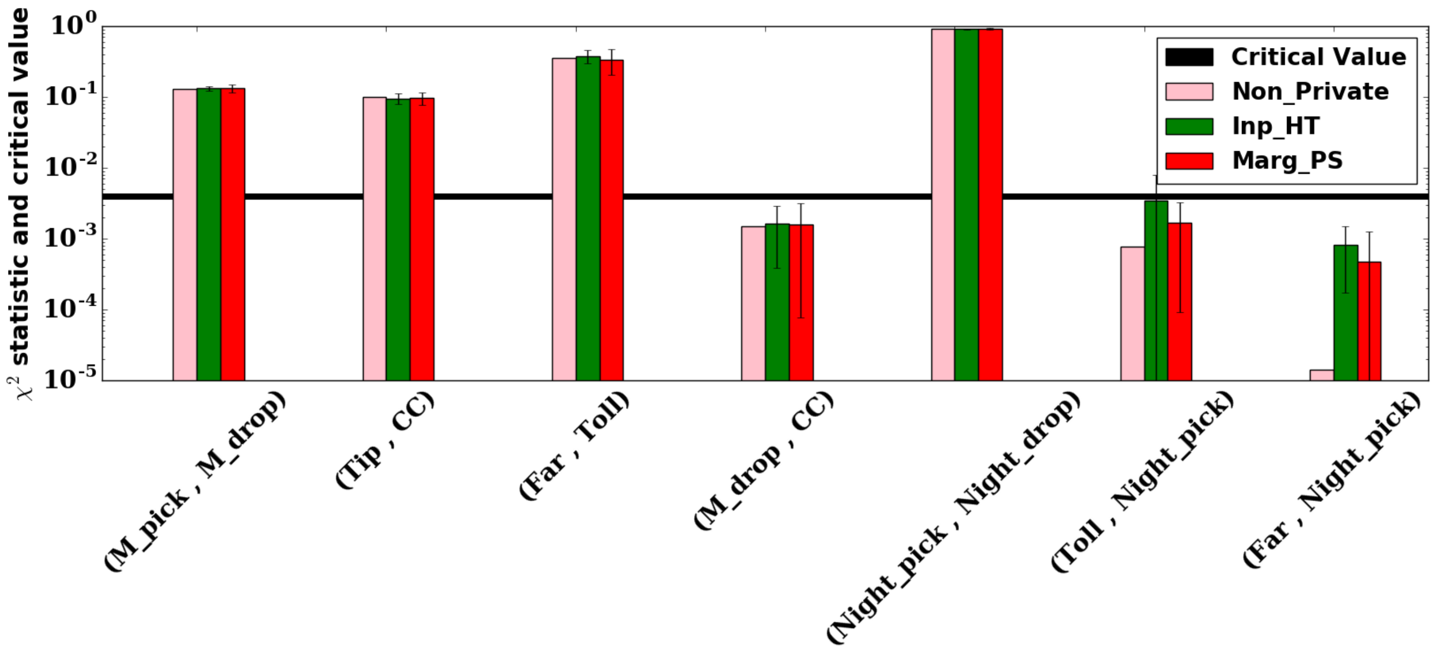
Empirical behaviour



- Compare three methods: Hadamard based (Inp_HT), marginal materialization (Marg_PS), Expectation maximization (Inp_EM)
- Measure sum of absolute error in materializing 2-way marginals
- N = 0.5M individuals, vary privacy parameter ε from 0.4 to 1.4

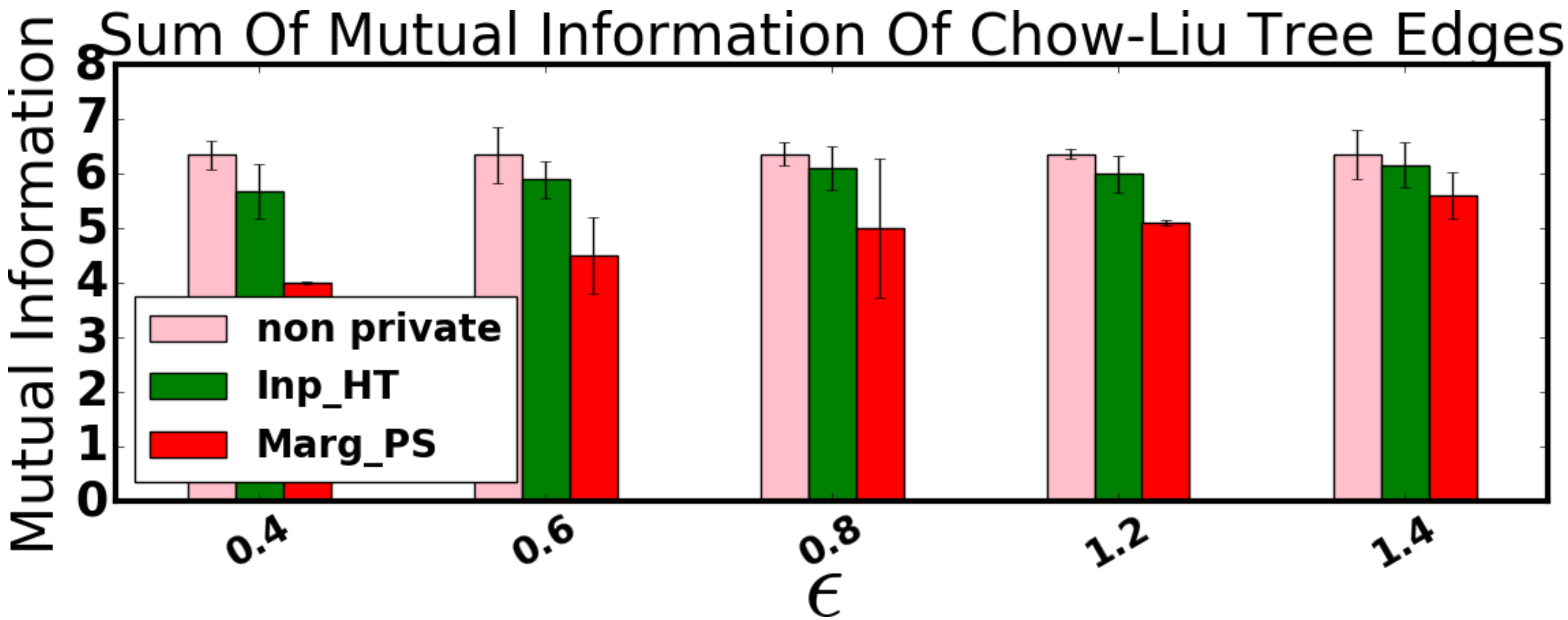
THE UNIVERSITY OF WARWICK

Applications – χ-squared test



- Anonymized, binarized NYC taxi data
- Compute χ-squared statistic to test correlation
- Want to be same side of the line as the non-private value!

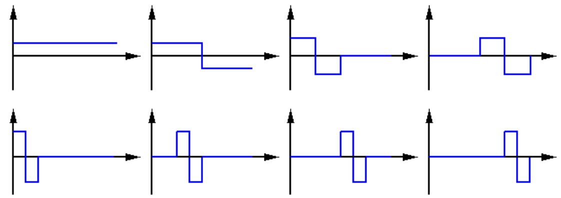
Application – building a Bayesian model

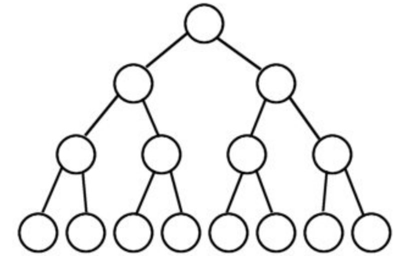


- **Aim:** build the tree with highest mutual information (MI)
- Plot shows MI on the ground truth data for evaluation purposes

THE UNIVERSITY OF WARWICK

Range Queries



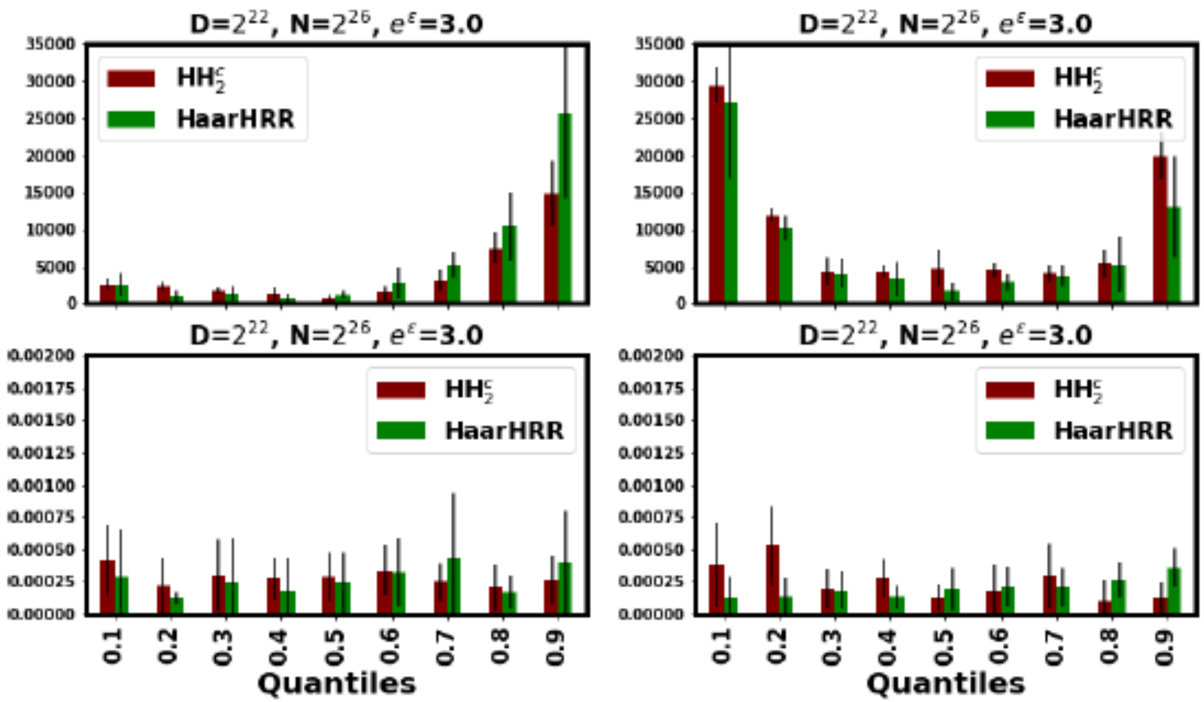


- Given data from an ordered domain, we study range queries:
  - "How many data points fall in the range [l, r]"?
- Hierarchical approaches improve over summing point queries:
  a) Impose a regular tree over the input domain, and sample nodes
     - Need to do post-processing to obtain consistent answers
  b) Apply a Haar wavelet transform to input, and sample coefficients
- Which method is best? Answer: both are competitive!
  - Similar variance (up to leading constant) for optimal settings
  - Similar empirical performance, slight preferences for different ε
  - In contrast to the centralized case, where trees are preferred

THE UNIVERSITY OF WARWICK

Quantile queries

- Use range queries to find ranges that cover a given fraction
  - E.g. the median is the 0.5 quantile query
- Both Hierarchical Histograms (HH) and Haar wavelets obtain similar results: very accurate answers for N large enough



THE UNIVERSITY OF WARWICK

Conclusions

- Private data release is a confounding problem!
  - We haven't yet got it right consistently enough
  - The idea of "1 click privacy" is still a long way off
- Current privacy work gives some cause for optimism
  - Statistical privacy, safety in numbers, and massive deployments
- Lots of opportunity for new work:
  - Designing optimal mechanisms for local differential privacy
  - Extend beyond simple counts and marginals
  - Structured data: graphs, movement patterns
  - Unstructured data: text, images, video?

Joint work with Divesh Srivastava (AT&T), Tejas Kulkarni (Warwick)

Supported by AT&T, Royal Society, European Commission

THE UNIVERSITY OF WARWICK